WILLIAM D. LAW, JR. LEARNING COMMONS

Matrix to Reduced Echelon Form, Step-by-Step

This handout accompanies the YouTube video - **https://www.youtube.com/watch?v=7piTfgtNa_U**

System of 3 equations

$2x + 5y + 4z = 12$

$3x + y - 4z = 1$

$x - 2y - 3z = 0$

Create the matrix.

$$\left[\begin{array}{cccc} 2 & 5 & 4 & 12 \\ 3 & 1 & -4 & 1 \\ 1 & -2 & -3 & 0 \end{array}\right]$$

The goal:

$$\left[\begin{array}{cccc} 1 & 0 & 0 & ? \\ 0 & 1 & 0 & ? \\ 0 & 0 & 1 & ? \end{array}\right]$$

Organize your work like this to show step-by-step how you put a matrix in reduced row echelon form.

Updated matrix	Plan for this step	Calculator step
$\left[\begin{array}{cccc} 2 & 5 & 4 & 12 \\ 3 & 1 & -4 & 1 \\ 1 & -2 & -3 & 0 \end{array}\right]$	$R_1 \leftrightarrow R_3$	MATRIX >MATH >**C:rowSwap([A],1,3)** Verify output is correct STO→[A]
$\left[\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 3 & 1 & -4 & 1 \\ 2 & 5 & 4 & 12 \end{array}\right]$	$-3(R_1) + R_2 \rightarrow R_2$ $\begin{array}{cccc} -3 & 6 & 9 & 0 \\ 3 & 1 & -4 & 1 \\ \hline 0 & 7 & 5 & 1 \end{array}$	MATRIX >MATH >**F:*row+(-3,[A],1,2)** Verify output is correct STO→[A]
$\left[\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & 7 & 5 & 1 \\ 2 & 5 & 4 & 12 \end{array}\right]$	$-2(R_1) + R_3 \rightarrow R_3$ $\begin{array}{cccc} -2 & 4 & 6 & 0 \\ 2 & 5 & 4 & 12 \\ \hline 0 & 9 & 10 & 12 \end{array}$	MATRIX >MATH >**E:*row+(-2,[A],1,3)** Verify output is correct STO→[A]
$\left[\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & 7 & 5 & 1 \\ 0 & 9 & 10 & 12 \end{array}\right]$	$1/7(R_2) \rightarrow R_2$	MATRIX > MATH > **F:*row(1/7,[A],2)** Verify output is correct STO→[A]
$\left[\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & 1 & .714 & .142 \\ 0 & 9 & 10 & 12 \end{array}\right]$	Change decimals to fractions	MATH > ENTER > ENTER Verify output is correct STO→[A]

Updated matrix	Plan for this step	Calculator step

$2(R_2)+R_1 \to R_1$

$$\left|\begin{array}{cccc} 1 & -2 & -3 & 0 \\ 0 & 1 & 5/7 & 1/7 \\ 0 & 9 & 10 & 12 \end{array}\right| \qquad \begin{array}{cccc} 0 & 2 & 10/7 & 2/7 \\ 1 & -2 & -3 & 0 \\ \hline 1 & 0 & -11/7 & 2/7 \end{array}$$

MATRIX > MATH > **F:*row+(2,[A],2,1)**

MATH > ENTER > ENTER

Verify output is correct. STO→[A]

$-9(R_2)+R_3 \to R_3$

$$\left|\begin{array}{cccc} 1 & 0 & -11/7 & 2/7 \\ 0 & 1 & 5/7 & 1/7 \\ 0 & 9 & 10 & 12 \end{array}\right| \qquad \begin{array}{cccc} 0 & -9 & -45/7 & -9/7 \\ 0 & 9 & 10 & 12 \\ \hline 0 & 0 & 25/7 & 75/7 \end{array}$$

MATRIX > MATH > **F:*row+(-9,[A],2,3)**

MATH > ENTER > ENTER

Verify output is correct. STO→[A]

$$\left|\begin{array}{cccc} 1 & 0 & -11/7 & 2/7 \\ 0 & 1 & 5/7 & 1/7 \\ 0 & 0 & 25/7 & 75/7 \end{array}\right|$$

$7/25(R_3) \to R_3$

MATRIX > MATH > **E:*row(7/25,[A],3)**

MATH > ENTER > ENTER

Verify output is correct. STO→[A]

$11/7(R_3)+R_1 \to R_1$

$$\left|\begin{array}{cccc} 1 & 0 & -11/7 & 2/7 \\ 0 & 1 & 5/7 & 1/7 \\ 0 & 0 & 1 & 3 \end{array}\right| \qquad \begin{array}{cccc} 0 & 0 & 11/7 & 33/7 \\ 1 & 0 & -11/7 & 2/7 \\ \hline 1 & 0 & 0 & 5 \end{array}$$

MATRIX >MATH >**F:*row+(11/7,[A],3,1)**

MATH > ENTER > ENTER

Verify output is correct. STO→[A]

$-5/7(R_3)+R_2 \to R_2$

$$\left|\begin{array}{cccc} 1 & 0 & 0 & 5 \\ 0 & 1 & 5/7 & 1/7 \\ 0 & 0 & 1 & 3 \end{array}\right| \qquad \begin{array}{cccc} 0 & 0 & -5/7 & -15/7 \\ 0 & 1 & 5/7 & 1/7 \\ \hline 0 & 1 & 0 & -2 \end{array}$$

MATRIX >MATH > **F:*row+(-5/7,[A],3,2)**

MATH > ENTER > ENTER

Verify output is correct. STO→[A]

The matrix is now in reduced-row-echelon form. Rewrite the system of equations and simplify.

$$\left|\begin{array}{cccc} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right| \qquad \begin{array}{lll} 1x+0y+0z=5 & \to & x=5 \\ 0x+1y+0z=-2 & \to & y=-2 \\ 0x+0y+1z=3 & \to & z=3 \end{array}$$

Write answer as an ordered triplet. $(5,-2,3)$