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LEARNING **COMMONS**

Using the Product Rule to find the Derivative

The Process for the Product Rule:

1. Given: $H(x) = f(x) \cdot g(x)$; then
2. Identify $f(x), f'(x), g(x), and\ g'(x)$;
3. $H'(x) = \frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$; Product rule
4. Substitute and simplify

Some other helpful rules:

$\frac{d}{dx}(c) = 0$ $\quad \frac{d}{dx}[cf(x)] = c\,f'(x)$ $\quad \frac{d}{dx}(x^n) = nx^{n-1}dx$ $\quad \frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}\ln|x| = \frac{1}{x}dx$ $\quad \frac{d}{dx}(a^x) = a^x \ln(a)$ $\quad \frac{d}{dx}(\sin x) = \cos x\,dx$ $\quad \frac{d}{dx}(\cos x) = -\sin x\,dx$

Algebraic Example of Product Rule:

$Given: H(x) = (x^2+1)(x^3+6)^7$

$f(x) = x^2 + 1$

$f'(x) = 2x$

$g(x) = (x^3 + 6)^7$

$g'(x) = 7(x^3+6)^6(3x^2) = 21x^2(x^3+6)^6$; *using the chain rule*

$H'(x) = f(x)g'(x) + g(x)f'(x)$ so, we have:

$H'(x) = (x^2+1)[21x^2(x^3+6)^6] + (x^3+6)^7(2x)$*; ask your teacher if you can stop here!*

Factor out like terms and then continue to simplify.

$H'(x) = (x^3+6)^6\,x[(x^2+1)(21x) + (x^3+6)(2)]$; distribute

$H'(x) = (x^3+6)^6\,x[21x^3 + 21x + 2x^3 + 12]$; combine like terms

$H'(x) = x(x^3+6)^6[23x^3 + 21x + 12]$

Combination of Terms Example Product Rule:

$Given: H(x) = x\ln(x) - x$ we will also use: $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

Term 1: $\boldsymbol{f(x)g(x)} = x\ln(x)$

$f(x) = x$

$f'(x) = 1$

$g(x) = \ln(x)$

$g^1(x) = \frac{1}{x}$

$H'(x) = f(x)g'(x) + g(x)f'(x)$ so, we have:

$H'(x) = x(\frac{1}{x}) + \ln(x) \cdot (1)$; then simplify

$H'(x) = 1 + \ln(x)$

$\boldsymbol{Term\ 2}$: $k(x) = -x$

$k(x) = -x$

$k'(x) = -1$

Combining the two pieces, our solution is: $H'(x) = 1 + \ln(x) - 1$; which simplifies to $\ln(x)$.

Algebraic Alternative to Product Rule:

$Given: H(x) = (x^2 + 1)(\, x^3 + 6)$
One alternate to this is to use algebra and foil the components together before taking the derivative.

$H(x) = (x^2 + 1)(\, x^3 + 6) = \; x^5 + 6x^2 + x^3 + 6$; then we can just use the power rule to take the derivative.

$H'(x) = 5x^4 + 12x + 3x^2$

Alternative to Quotient Rule Example:

$Given: H(x) = \frac{x^3+3}{(x+5)^2}$; Keeping in mind the exponent rule: $m^{-a} = \frac{1}{m^a}$, we can rewrite this expression.

$H(x) \; = (x^3 + 3)(x + 5)^{-2}$

$f(x) = \; x^3 + 3$ $\qquad g(x) = (x + 5)^{-2}$
$f'(x) = \; 3x^2$ $\qquad g'(x) = -2(x + 5)^{-3}$

$H'(x) = \frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$; ; so, we have:

$H'(x) = (x^3 + 3)[-2(x + 5)^{-3}] + \; (x + 5)^{-2}(3x^2)$; then we begin simplifying if necessary

$H'(x) = (x + 5)^{-3}[-2(x^3 + 3) + (x + 5)\,(3x^2)]$; factoring out like terms

$H'(x) = (x + 5)^{-3}[(-2x^3 - 6) + (3x^3 + 15x^2)]$; combining like terms

$H'(x) = (x + 5)^{-3}[-6 + 2x^3 + 15x^2]$; one last re-write

$$H'(x) = \frac{-6 + 2x^3 + 15x^2}{(x + 5)^3}$$

You try's:

1. H(x) = $(x + 5)(x^2 + 1)^6$
2. H(x) = $x^2 e^{-3x}$
3. H(x) = $\sec(\theta)\tan(\theta)$

Solutions:

1. H'(x)= $(x^2 + 1)^5(13x^2 + 60x + 1)$
2. H'(x) = $xe^{-3x}(-3x + 2)$
3. H'(x) = $sec^3(\theta) + \sec(\theta)tan^2(\theta)$ or $\sec(\theta)\,[2tan^2(\theta) + 1]$