WILLIAM D. LAW, JR. LEARNING COMMONS

Tests for Convergence of Series

Test for Divergence

$$\text{If } \lim_{n\to\infty} a_n \neq 0 \text{ or fails to exist, then } \sum_{n=1}^{\infty} a_n \text{ diverges.}$$

Geometric Series

$$\sum_{n=1}^{\infty} ar^{n-1} \quad \text{converges for } |r| < 1,\ s = \frac{a}{1-r}$$

Diverges for $|r| \geq 1$

p Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \qquad \text{converges for } p > 1$$

Diverges for $p \leq 1$

Ratio Test

$$\text{If} \quad \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$$

Root Test

$$\text{If} \quad \lim_{n\to\infty} \sqrt[n]{|a_n|} = L$$

i. The series is Absolutely convergent if L < 1 and therefore is convergent
ii. The series diverges if L > 1 or is infinite
iii. The test is inconclusive if L = 1

Absolute Convergence

$$\text{If} \quad \sum_{n=1}^{\infty} |a_n| \quad \text{converges, then} \quad \sum_{n=1}^{\infty} a_n \text{ converges absolutely and is convergent.}$$

Integral Test

Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$.

i) If $\int_1^{\infty} f(x)\,dx$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent

ii) If $\int_1^{\infty} f(x)\,dx$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.

The Limit Comparison Test

Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms.

$$\text{If} \quad \lim_{n\to\infty} \frac{a_n}{b_n} = c$$

where c is a finite number and $c > 0$ then both series converge or both series diverge.

The Comparison Test

Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms.

i) If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent.

i) If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n, then $\sum a_n$ is also divergent.

Alternating Series Test

If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \cdots \qquad b_n > 0$ satisfies

i) $b_{n+1} \leq b_n \quad \forall n \qquad$ ii) $\lim_{n\to\infty} b_n = 0 \qquad$ then the series is convergent.

This handout was Prepare by TCC instructor Rhonda MacLeod, condensed from chapter 11, Stewart's Calculus, 7e