# Composite Functions - Practice (and solutions)

For the given functions *f* and *g*, find (answer on the back)

(a) $(f \circ g)(x)$ (b) $(g \circ f)(x)$ (c) $(f \circ f)(x)$ (d) $(g \circ g)(x)$

1. $f(x) = 2x + 3, \quad g(x) = 3x$

2. $f(x) = \sqrt{x}, \quad g(x) = x^2$

3. $f(x) = \dfrac{x+1}{x-1}, \quad g(x) = \dfrac{x-1}{x+1}$

4. $f(x) = x + \dfrac{1}{x}, \quad g(x) = x^2$

For each of the following problems, show that $(f \circ g)(x) = (g \circ f)(x) = x$.

1. $f(x) = 2x, \quad g(x) = \frac{1}{2}x$

2. $f(x) = ax + b, \quad g(x) = \frac{1}{a}(x - b), \quad a \neq 0$

3. $f(x) = \dfrac{1}{x}, \quad g(x) = \dfrac{1}{x}$

4. $f(x) = \dfrac{2x + 1}{x - 1}, \quad g(x) = \dfrac{x + 1}{x - 2}$

Answers

1 a) $f(g(x)) = 2(3x) + 3 = 6x + 3$

b) $g(f(x)) = 3(2x + 3) = 6x + 9$

c) $f(f(x)) = 2(2x + 3) + 3 = 4x + 9$

d) $g(g(x)) = 3(3x) = 9x$

2 a) $f(g(x)) = \sqrt{(x^2)} = x$

b) $g(f(x)) = (\sqrt{x})^2 = x$

c) $f(f(x)) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$

d) $g(g(x)) = (x^2)^2 = x^4$

3 a) $f(g(x)) = \dfrac{\left(\frac{x-1}{x+1}\right)+1}{\left(\frac{x-1}{x+1}\right)-1} = -x$

b) $g(f(x)) = \dfrac{\left(\frac{x+1}{x-1}\right)-1}{\left(\frac{x+1}{x-1}\right)+1} = \dfrac{1}{x}$

c) $f(f(x)) = \dfrac{\left(\frac{x+1}{x-1}\right)+1}{\left(\frac{x+1}{x-1}\right)-1} = x$

d) $g(g(x)) = \dfrac{\left(\frac{x-1}{x+1}\right)-1}{\left(\frac{x-1}{x+1}\right)+1} = -\dfrac{1}{x}$

4 a) $f\left(g(x)\right) = (x^2) + \frac{1}{(x^2)} = \frac{x^4+1}{x^2}$

b) $g\left(f(x)\right) = \left(x + \frac{1}{x}\right)^2 = \frac{x^4+2x^2+1}{x^2}$

c) $f\left(f(x)\right) = \left(x + \frac{1}{x}\right) + \frac{1}{\left(x+\frac{1}{x}\right)} = \frac{x^4+3x^2+1}{x^3+x}$

d) $g\left(g(x)\right) = (x^2)^2 = x^4$