Slope-Intercept Form and Point-Slope Form

Slope of the line	$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$
Slope-Intercept Form	$y = mx + b$ $\quad$ *m is slope; b is y-intercept*
Point-Slope Form	$y = m(x - x_1) + y_1$ or $\quad y - y_1 = m(x - x_1)$
Slope of parallel lines	$m_1 = m_2$ (slopes are the same)
Slope of perpendicular lines	$m_1 m_2 = -1$ (slopes are opposite & reciprocal)
Equations of Horizontal and Vertical Lines	$y = b$ horizontal line $x = a$ vertical line, where a & b are constants

Example (1): Write the slope - intercept equation of a line which passes through (0,-7) whose slope is 2.

Solution:

Slope-intercept equation is $y = mx + b$**.** What we need to complete this equation are slope (m) & y-intercept $(0, b)$, and the problem provides both information.

$m = 2$ and $b = -7$ The equation of the line is $y = 2x - 7$

Example (2): Write the slope-intercept equation of a line which passes through $\overset{x_1, y_1}{(0,4)}$ and $\overset{x_2, y_2}{(3,-5)}$.

Solution:

Slope-intercept equation is $y = mx + b$**.** What we need to complete this equation are slope (m) & y-intercept $(0, b)$, however, we only have y-intercept.

To find the slope, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 4}{3 - 0} = \frac{-9}{3} = -3$

$m = -3$ and $b = 4$ The equation of the line is $y = -3x + 4$

Example (3): Write the slope-intercept equation of a line which passes through $\overset{x_1, y_1}{(-1,4)}$ whose slope is 5.

Solution:

Slope-intercept equation is $y = mx + b$**.** What we need to complete this equation are slope (m) & y-intercept $(0, b)$, however, we only have slope. Here there are two ways to find the equation of the line.

Method I We will substitute m and (x_1, y_1) in the form $y = mx + b$ to solve for b.

$m = 5$, $(x_1, y_1) = (-1,4)$ $\qquad 4 = 5(-1) + b$

$\Rightarrow \quad b = 9$

The equation of the line is $y = 5x + 9$

Method II Since we are given slope m and an ordered pair (x_1, y_1), we can use Point-slope form to find equation of the line.

Point-slope form is $y = m(x - x_1) + y_1$ $\qquad y = 5(x - (-1)) + 4$

$m = 5$, $(x_1, y_1) = (-1,4)$ $\qquad \Rightarrow \quad y = 5(x + 1) + 4$ Simplify the parenthesis

$\Rightarrow \quad y = 5x + 5 + 4$ Distribute 5 into parenthesis

$\Rightarrow \quad y = 5x + 9$

Example (4): Write the slope-intercept equation of a line which passes through $\overset{x_1, y_1}{(1,3)}$ and $\overset{x_2, y_2}{(-5,-1)}$.

Solution:

Slope-intercept equation is $y = mx + b$**.** What we need to complete this equation are slope (m) & y-intercept $(0, b)$. However, we are given two ordered pairs (x_1, y_1) and (x_2, y_2) without slope and y-intercept. Therefore, we need to find the slope first. Then we can use the two methods discussed on Example (3) to find the equation of the line.

(x_1, y_1) (x_2, y_2)

To find the slope between two ordered pairs, $(1,3)$ and $(-5,-1)$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{-5-1} = \frac{-4}{-6} = \frac{2}{3}$

Method I Now we have slope, we will substitute m and (x_1, y_1) in the form $y = mx + b$ to solve for b.

$m = \frac{2}{3}$, $(x_1, y_1) = (1,3)$ $\quad 3 = \frac{2}{3}(1) + b$

$\Rightarrow \quad b = \frac{7}{3}$

The equation of the line is $y = \frac{2}{3}x + \frac{7}{3}$

Method II We also can use Point-slope form to find the equation of the line.

Point-slope form is $y = m(x - x_1) + y_1$

$m = \frac{2}{3}$, $(x_1, y_1) = (1,3)$

$y = \frac{2}{3}(x-1) + 3$

$\Rightarrow \quad y = \frac{2}{3}x - \frac{2}{3} + 3$ Distribute $\frac{2}{3}$ into parenthesis

$\Rightarrow \quad y = \frac{2}{3}x - \frac{2}{3} + \frac{9}{3}$ Combine like term

$\Rightarrow \quad y = \frac{2}{3}x + \frac{7}{3}$

Example (5): Write the slope-intercept equation of a line which is parallel to $y = 4x - 2$, passing through $(1,3)$.
x_1, y_1

Solution:

Slope-intercept equation is $y = mx + b$**.** What we need to complete this equation are slope (m) & y-intercept $(0, b)$. Since the line we're looking for is **parallel** to $y = 4x - 2$**, their slopes are the same**, $m = 4$.

Method I We will substitute m and (x_1, y_1) in the form $y = mx + b$ to solve for b.

$m = 4$, $(x_1, y_1) = (1,3)$ $\quad 3 = 4(1) + b \quad \Rightarrow \quad b = -1$

The equation of the line is $y = 4x - 1$

Method II We also can use Point-slope form to find the equation of the line.

Point-slope form is $y = m(x - x_1) + y_1$ $\quad y = 4(x-1)+3$

$m = 4$, $(x_1, y_1) = (1,3)$ $\quad$ => $y = 4x - 4 + 3$ Distribute 4 into parenthesis

=> $y = 4x - 1$ Combine like term

Example (6): Write the slope-intercept equation of a line which is perpendicular to $y = -\frac{1}{3}x + 4$, passing through $\overset{x_1, y_1}{(-3,5)}$.

Solution:

Slope-intercept equation is $y = mx + b$**.** What we need to complete this equation are the slope (m)& y-intercept $(0, b)$. Since our line is **perpendicular** to $y = -\frac{1}{3}x + 4$ **(**which was given**),** we can find the slope of our line by **taking the opposite sign and using the reciprocal of the given line** which has a slope of $m = -\frac{1}{3}$. Therefore, **the slope of our line is** $m = 3$ (the perpendicular one to the given line)

Method I We will substitute m and (x_1, y_1) in the form $y = mx + b$ to solve for b.

$m = 3$, $(x_1, y_1) = (-3,5)$ $\quad 5 = 3(-3) + b$

=> $5 = -9 + b$

=> $b = 14$

The equation of the line is $y = 3x + 14$

Method II We also can use Point-slope form to find the equation of the line.

Point-slope form is $y = m(x - x_1) + y_1$ $\quad y = 3(x - (-3)) + 5$

$m = 3$, $(x_1, y_1) = (-3,5)$ $\quad$ => $y = 3(x + 3) + 5$ Simplify the parenthesis

=> $y = 3x + 9 + 5$ Distribute 5 into parenthesis

=> $y = 3x + 14$

Example (7): Write an equation of a vertical line which passes through $\overset{x_1, y_1}{(-1,6)}$.

Solution:

The equation of a vertical line is $x = a$

The *x*-coordinate of the point $(-1,6)$ is -1. Therefore, the equation of the vertical line is $x = -1$

Example (8): Write an equation of a horizontal line which passes through $\overset{x_1, y_1}{\left(\frac{3}{4},-\frac{5}{6}\right)}$.

Solution:

The equation of a horizontal line is $y = b$

The *y*-coordinate of the point $\left(\frac{3}{4},-\frac{5}{6}\right)$ is $-\frac{5}{6}$. Therefore, the equation of the horizontal line is $y = -\frac{5}{6}$

Exercises:

1. Write the slope - intercept equation of a line which passes through (0,5) whose slope is 4.

2. Write the slope-intercept equation of a line which passes through $(0,-3)$ and $(4,5)$.

3. Write the slope-intercept equation of a line which passes through $(4,0)$ and $(7,-1)$.

4. Write the slope-intercept equation of a line which is parallel to $y = 3x + 5$, passing through $(-6,3)$

5. Write the slope-intercept equation of a line which is perpendicular to $y = 7x + 2$, passing through $(3,2)$

6. Write an equation of a horizontal line which passes through $(5,-1)$

7. Write an equation of a vertical line which passes through $\left(8, \frac{7}{3}\right)$.

Answers:

1. $y = 4x + 5$ **2.** $y = 2x - 3$ **3.** $y = -\frac{1}{3}x + \frac{4}{3}$ **4.** $y = 3x + 21$ **5.** $y = -\frac{1}{7}x + \frac{17}{7}$

6. $y = -1$ **7.** $x = 8$