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LEARNING COMMONS

Operations on Rational Numbers (Fractions)

Rational Numbers are numbers which can be written as a ratio of two integers. This includes all numbers which are obviously fractions, such as $-\frac{2}{3}$, $\frac{5}{9}$, and $-\frac{24}{7}$, as well as integers, for example $\frac{-4}{1}$ and $\frac{3}{1}$, and all terminating and repeating decimals like 0.3 = $\frac{3}{10}$, $0.87\overline{5} = \frac{197}{225}$, 0.875 = $\frac{7}{8}$, or 0.6 = $\frac{2}{3}$.

Remember that any fraction can be written as a decimal by dividing the denominator into the numerator.

In operations with fractions, we have to remember all of the rules for working with fractions and all of the rules for working with signed numbers. In our examples we will work two problems with each operation and review the rules for signed numbers as we go.

EXAMPLE 1: $-\frac{3}{4}+\left(-\frac{7}{10}\right)$

Here we are adding two fractions which have the same sign. They are both negative numbers.

> REMEMBER the rule:
>
> If you are adding and the numbers both have the same sign, add the absolute values and attach the common sign.

This means that the sum will have the same sign as the two fractions we are adding together.

$-\frac{3}{4}+\left(-\frac{7}{10}\right)$ We will need to find the least common denominator. LCD = 20

$= -\frac{3}{4}\cdot\frac{5}{5}+\left(-\frac{7}{10}\right)\cdot\frac{2}{2}$ Then we will need to build equivalent fractions.

$= \dfrac{-15+(-14)}{20} = -\dfrac{29}{20}$ We can add the numerators and attach the common sign.

$= -1\dfrac{9}{20}$ We could also change the improper fraction to a mixed number. *Both* of the original fractions were negative, so the answer was negative.

EXAMPLE 2: $-\dfrac{7}{15} + \dfrac{9}{5}$

Here the signs are different. We have one positive number and one negative number.

REMEMBER the rule:

If you are adding and the **signs are *different***, find the *difference* between the absolute values. The sum will have the same sign as the number with the greater absolute value.

It is not always easy to tell with fractions which one has the greater absolute value. For this reason we will first find the LCD and build equivalent fractions. We can then apply the rule to the numerators of the fractions.

$-\dfrac{7}{15}+\dfrac{9}{5}$ We will need to find the least common denominator.

$= -\dfrac{7}{15}+\dfrac{9}{5}\cdot\dfrac{3}{3}$ We will need to build equivalent fractions.

$= \dfrac{-7+27}{15}$ We can now apply the rule for different signs to the numerators.

$= \dfrac{20}{15}$ The sign is <u>positive</u>.

$= \dfrac{4}{3}$ Reduce the fraction. As the number with the greater absolute value was positive in this problem the answer is also positive.

EXAMPLE 3: $\frac{7}{12} - \frac{3}{4}$

> REMEMBER the rule for subtraction:
>
> First rewrite the subtraction as addition of the opposite number. Then follow the rules for addition.

$\frac{7}{12} - \frac{3}{4} = \frac{7}{12} + \left(-\frac{3}{4}\right)$ Now we have an addition problem similar to the last one. The signs are different.

$\frac{7}{12} + \left(-\frac{3}{4}\right)$ Find the LCD

$= \frac{7}{12} + \left(-\frac{3}{4} \cdot \frac{3}{3}\right)$ Build equivalent fractions.

$= \frac{7 + (-9)}{12} = -\frac{2}{12}$ Add the numerators. The sign is <u>negative</u>.

$= -\frac{1}{6}$ Reduce the fraction. The answer stays negative.

EXAMPLE 4: $\frac{5}{6} - \left(-\frac{2}{9}\right)$

First, rewrite the subtraction as addition of the opposite number.

$\frac{5}{6} - \left(-\frac{2}{9}\right) = \frac{5}{6} + \frac{2}{9}$ You can see that we are now adding two positive fractions.

$\frac{5}{6} + \frac{2}{9}$ Find the LCD

$\frac{5}{6} \cdot \frac{3}{3} + \frac{2}{9} \cdot \frac{2}{2} = \frac{15 + 4}{18}$ Build the equivalent fractions and add.

$= \frac{19}{18} = 1\frac{1}{18}$ Add the numerators and change to a mixed number.

EXAMPLE 5: $-\frac{8}{15}\cdot\left(-\frac{5}{12}\right)$ Multiplying fractions with the same sign.

> REMEMBER the rule:
>
> If you are multiplying and the signs are the same, product will *always* be *positive*.

$-\frac{8}{15}\cdot\left(-\frac{5}{12}\right)$ Signs are the same, so the answer will be positive.

$= \frac{8}{15}\cdot\frac{5}{12}$ Find the prime factorization of each factor.

$= \frac{\overset{1}{\cancel{2}}\cdot\overset{1}{\cancel{2}}\cdot 2\cdot\overset{1}{\cancel{5}}}{3\cdot\underset{1}{\cancel{5}}\cdot\underset{1}{\cancel{2}}\cdot\underset{1}{\cancel{2}}\cdot 3}$ Divide out (cancel) common factors and multiply the remaining factors.

$= \frac{2}{9}$

EXAMPLE 6: $-\frac{17}{3}\cdot\frac{9}{2}$ Multiplying with opposite signs.

> REMEMBER the rule:
>
> If you are multiplying and the signs are different the product will *always* be *negative*.

$= -\frac{17}{9}\cdot\frac{9}{2}$ Signs are different, so the answer will be negative.

$= -\frac{17}{3}\cdot\frac{9}{2}$ Find the prime factorization of each factor.

$= -\frac{17\cdot\overset{1}{\cancel{3}}\cdot 3}{\underset{1}{\cancel{3}}\cdot 2}$ Divide out (cancel) common factors and multiply the remaining factors.

$= -\frac{51}{2} = -25\frac{1}{2}$ Change to a mixed number.

EXAMPLE 7: $-\frac{9}{16} \div \left(-\frac{3}{8}\right)$ Dividing fractions with the same sign.

> REMEMBER the rule:
>
> If you are dividing and the signs are the same, the quotient will *always* be *positive*.

$-\frac{9}{16} \div \left(-\frac{3}{8}\right)$ The signs are the same, so the answer is positive.

$= -\frac{9}{16} \cdot \left(-\frac{8}{3}\right)$ Change the operation to multiplication and invert the divisor. Notice the reciprocal of a negative number is also negative.

$= \frac{3 \cdot \cancel{3}^{1} \cdot \cancel{2}^{1} \cdot \cancel{2}^{1} \cdot \cancel{2}^{1}}{\cancel{2}_{1} \cdot \cancel{2}_{1} \cdot \cancel{2}_{1} \cdot 2 \cdot \cancel{3}_{1}}$ Find the prime factorization and cancel.

$= \frac{3}{2}$ or $1\frac{1}{2}$

EXAMPLE 8: $\frac{11}{3} \div \left(-\frac{22}{9}\right)$ Dividing with opposite signs.

> REMEMBER the rule:
>
> If you are dividing and the signs are different the quotient will <u>always</u> be negative.

$= \frac{11}{3} \cdot \left(-\frac{9}{22}\right)$ The signs are different, the answer is negative.

$= -\frac{\cancel{11}^{1} \cdot 3 \cdot \cancel{3}^{1}}{\cancel{3}_{1} \cdot 2 \cdot \cancel{11}_{1}}$ Find the prime factorization and cancel.

$= -\frac{3}{2} = -1\frac{1}{2}$ Change to a mixed number

EXERCISES

1. $\frac{1}{9}-\frac{5}{27}$

2. $-\frac{7}{12}+\frac{5}{8}$

3. $-\frac{5}{6}-\frac{5}{9}$

4. $\frac{1}{3}-\frac{1}{4}-\frac{1}{5}$

5. $\frac{5}{16}+\frac{1}{8}-\frac{1}{2}$

6. $-\frac{2}{9}\cdot\left(-\frac{3}{14}\right)$

7. $-\frac{6}{11}\div\frac{4}{9}$

8. $\frac{1}{8}\div\left(-\frac{5}{12}\right)$

9. $\frac{5}{8}\cdot\left(-\frac{7}{12}\right)\cdot\frac{16}{25}$

10. $\frac{5}{12}\cdot\frac{8}{15}\cdot\left(-\frac{1}{3}\right)$

KEY:

1. $-\frac{2}{27}$ 2. $\frac{1}{24}$ 3. $-\frac{25}{18}$ or $-1\frac{7}{18}$ 4. $-\frac{7}{60}$ 5. $-\frac{1}{16}$

6. $\frac{1}{21}$ 7. $-\frac{27}{22}$ or $-1\frac{5}{22}$ 8. $-\frac{3}{10}$ 9. $-\frac{7}{30}$ 10. $-\frac{2}{27}$