Slope-Intercept Form and Point-Slope Form

Slope of the line	$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$
Slope-Intercept Form	$y = mx + b$ *m is slope; b is y-intercept*
Point-Slope Form	$y = m(x - x_1) + y_1$ or $y - y_1 = m(x - x_1)$
Slope of parallel lines	$m_1 = m_2$ (slopes are the same)
Slope of perpendicular lines	$m_1 m_2 = -1$ (slopes are opposite & reciprocal)
Equations of Horizontal and Vertical Lines	$y = b$ horizontal line $x = a$ vertical line, where a & b are constants

Example (1): Write the slope - intercept equation of a line which passes through (0,-7) whose slope is 2.

Solution:

Slope-intercept equation is $y = mx + b$. What we need to complete this equation are <u>slope</u> (m) & <u>y-intercept</u> $(0, b)$, and the problem provides both information.

$m = 2$ and $b = -7$ The equation of the line is $y = 2x - 7$

Example (2): Write the slope-intercept equation of a line which passes through $\overset{x_1,\, y_1}{(0,4)}$ and $\overset{x_2,\, y_2}{(3,-5)}$.

Solution:

Slope-intercept equation is $y = mx + b$. What we need to complete this equation are <u>slope</u> (m) & <u>y-intercept</u> $(0, b)$, however, we only have <u>y-intercept</u>.

To find the slope, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-4}{3-0} = \frac{-9}{3} = -3$

$m = -3$ and $b = 4$ The equation of the line is $y = -3x + 4$

Example (3): Write the slope-intercept equation of a line which passes through $\overset{x_1,\,y_1}{(-1,4)}$ whose slope is 5.

Solution:

Slope-intercept equation is $y = mx + b$**.** What we need to complete this equation are slope (m) & y-intercept $(0, b)$, however, we only have slope. Here there are two ways to find the equation of the line.

$\boxed{\text{Method I}}$ We will substitute m and (x_1, y_1) in the form $y = mx + b$ to solve for b.

$m = 5$, $(x_1, y_1) = (-1,4)$ $\qquad 4 = 5(-1) + b$

=> $b = 9$

The equation of the line is $y = 5x + 9$

$\boxed{\text{Method II}}$ Since we are given slope m and an ordered pair (x_1, y_1), we can use Point-slope form to find equation of the line.

Point-slope form is $y = m(x - x_1) + y_1$ $\qquad y = 5(x - (-1)) + 4$

$m = 5$, $(x_1, y_1) = (-1,4)$ $\qquad$ => $y = 5(x + 1) + 4$ Simplify the parenthesis

=> $y = 5x + 5 + 4$ Distribute 5 into parenthesis

=> $y = 5x + 9$

Example (4): Write the slope-intercept equation of a line which passes through $\overset{x_1,\,y_1}{(1,3)}$ and $\overset{x_2,\,y_2}{(-5,-1)}$.

Solution:

Slope-intercept equation is $y = mx + b$**.** What we need to complete this equation are slope (m) & y-intercept $(0, b)$. However, we are given two ordered pairs (x_1, y_1) and (x_2, y_2) without slope and y-intercept. Therefore, we need to find the slope first. Then we can use the two methods discussed on Example (3) to find the equation of the line.

To find the slope between two ordered pairs, $\underset{(x_1, y_1)}{(1,3)}$ and $\underset{(x_2, y_2)}{(-5,-1)}$ $\quad m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-3}{-5-1} = \dfrac{-4}{-6} = \dfrac{2}{3}$

$\boxed{\text{Method I}}$ Now we have slope, we will substitute m and (x_1, y_1) in the form $y = mx + b$ to solve for b.

$m = \dfrac{2}{3}$, $(x_1, y_1) = (1,3)$ $\qquad 3 = \dfrac{2}{3}(1) + b$

$\Rightarrow \quad b = \dfrac{7}{3}$

The equation of the line is $y = \dfrac{2}{3}x + \dfrac{7}{3}$

$\boxed{\text{Method II}}$ We also can use Point-slope form to find the equation of the line.

Point-slope form is $y = m(x - x_1) + y_1$

$m = \dfrac{2}{3}$, $(x_1, y_1) = (1,3)$

$y = \dfrac{2}{3}(x-1) + 3$

$\Rightarrow \quad y = \dfrac{2}{3}x - \dfrac{2}{3} + 3$ Distribute $\dfrac{2}{3}$ into parenthesis

$\Rightarrow \quad y = \dfrac{2}{3}x - \dfrac{2}{3} + \dfrac{9}{3}$ Combine like term

$\Rightarrow \quad y = \dfrac{2}{3}x + \dfrac{7}{3}$

Example (5): Write the slope-intercept equation of a line which is parallel to $y = 4x - 2$, passing through $\underset{x_1, y_1}{(1,3)}$.

Solution:

Slope-intercept equation is $y = mx + b$. What we need to complete this equation are <u>slope</u> (m) & <u>y-intercept</u> $(0, b)$. Since the line we're looking for is **parallel** to $y = 4x - 2$, **their slopes are the same**, $m = 4$.

$\boxed{\text{Method I}}$ We will substitute m and (x_1, y_1) in the form $y = mx + b$ to solve for b.

$m = 4$, $(x_1, y_1) = (1,3)$ $\qquad 3 = 4(1) + b \quad \Rightarrow \quad b = -1$

The equation of the line is $y = 4x - 1$

$\boxed{\text{Method II}}$ We also can use Point-slope form to find the equation of the line.

Point-slope form is $y = m(x - x_1) + y_1$

$m = 4$, $(x_1, y_1) = (1,3)$

$y = 4(x-1)+3$

=> $y = 4x - 4 + 3$ Distribute 4 into parenthesis

=> $y = 4x - 1$ Combine like term

Example (6): Write the slope-intercept equation of a line which is perpendicular to $y = -\frac{1}{3}x + 4$, passing through $\overset{x_1,\, y_1}{(-3,5)}$.

Solution:

Slope-intercept equation is $y = mx + b$**.** What we need to complete this equation are the slope (m) & y-intercept $(0, b)$. Since our line is **perpendicular** to $y = -\frac{1}{3}x + 4$ **(**which was given**)**, we can find the slope of our line by **taking the opposite sign and using the reciprocal of the given line** which has a slope of $m = -\frac{1}{3}$. Therefore, **the slope of our line is** $m = 3$ (the perpendicular one to the given line)

$\boxed{\text{Method I}}$ We will substitute m and (x_1, y_1) in the form $y = mx + b$ to solve for b.

$m = 3$, $(x_1, y_1) = (-3,5)$

$5 = 3(-3) + b$

=> $5 = -9 + b$

=> $b = 14$

The equation of the line is $y = 3x + 14$

$\boxed{\text{Method II}}$ We also can use Point-slope form to find the equation of the line.

Point-slope form is $y = m(x - x_1) + y_1$

$m = 3$, $(x_1, y_1) = (-3,5)$

$y = 3(x - (-3)) + 5$

=> $y = 3(x+3)+5$ Simplify the parenthesis

=> $y = 3x + 9 + 5$ Distribute 5 into parenthesis

=> $y = 3x + 14$

Example (7): Write an equation of a vertical line which passes through $\overset{x_1,\,y_1}{(-1,6)}$.

Solution:

The equation of a vertical line is $x = a$

The x-coordinate of the point $(-1,6)$ is -1. Therefore, the equation of the vertical line is $x = -1$

Example (8): Write an equation of a horizontal line which passes through $\overset{x_1,\,y_1}{\left(\frac{3}{4}, -\frac{5}{6}\right)}$.

Solution:

The equation of a horizontal line is $y = b$

The y-coordinate of the point $\left(\frac{3}{4}, -\frac{5}{6}\right)$ is $-\frac{5}{6}$. Therefore, the equation of the horizontal line is $y = -\frac{5}{6}$

Exercises:

1. Write the slope - intercept equation of a line which passes through (0,5) whose slope is 4.
2. Write the slope-intercept equation of a line which passes through $(0,-3)$ and $(4,5)$.
3. Write the slope-intercept equation of a line which passes through $(4,0)$ and $(7,-1)$.
4. Write the slope-intercept equation of a line which is parallel to $y = 3x + 5$, passing through $(-6,3)$
5. Write the slope-intercept equation of a line which is perpendicular to $y = 7x + 2$, passing through $(3,2)$
6. Write an equation of a horizontal line which passes through $(5,-1)$
7. Write an equation of a vertical line which passes through $\left(8, \frac{7}{3}\right)$.

Answers:

1. $y = 4x + 5$ **2.** $y = 2x - 3$ **3.** $y = -\frac{1}{3}x + \frac{4}{3}$ **4.** $y = 3x + 21$ **5.** $y = -\frac{1}{7}x + \frac{17}{7}$

6. $y = -1$ **7.** $x = 8$