Solving Inequalities in the Form $x + a < b$ and $ax < b$

Inequalities are different from equations (or equalities). The difference is in the solutions or solution sets.

If we restrict ourselves to linear equations and inequalities in one variable, we can compare the solutions in the following way.

$x = 2$ -- this is an equality

-- the left side of the equation has the **<u>same value</u>** as the right side.

-- x has only **<u>one</u>** value

-- x can only be equal to 2

$x > 2$ -- this is an **<u>inequality</u>**

-- the left side is greater than the right side

-- x has an **<u>infinite</u>** number of values

-- x can be any number greater than 2

$x \geq 2$ -- this is an **<u>inequality</u>**

-- the left side may be **<u>equal to</u>** the right side but it may also be greater than the right side

-- x has an **<u>infinite</u>** number of values

-- x can be equal to 2 or any number greater than 2

$x < 2$ -- this is an **<u>inequality</u>**

-- the left side is **<u>less than</u>** the right side

-- x has an **<u>infinite</u>** number of values

-- x can be any number less than 2

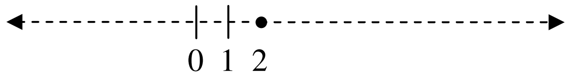
$x \leq 2$ -- this is an **<u>inequality</u>**

-- the left side may be **<u>equal to</u>** the right side or it may be **<u>less than</u>** the right side

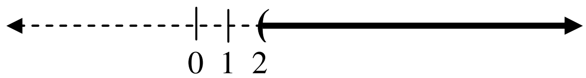
-- x has an **<u>infinite</u>** number of values

-- x can be equal to 2 or any number **<u>less than</u>** 2

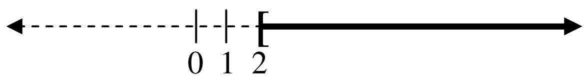
The solution sets are as follows:

$x = 2$  equality

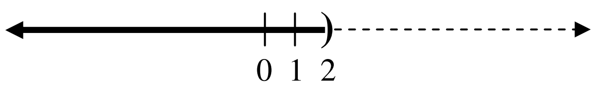
Solution set is 2, which is shown as one point on the number line.

$x > 2$ 

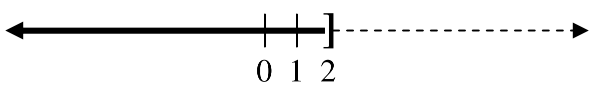
Solution set is all numbers greater than 2. Use  because 2 is not a solution.

$x \geq 2$ 

Solution set is 2 and all numbers greater than 2. Use  because 2 is a solution.

$x < 2$  inequalities

Solution set is all numbers less than 2. Use  because 2 is not a solution.

$x \leq 2$ 

Solution set is 2 and all numbers less than 2. Use  because 2 is a solution.

To graph the solution set of an inequality, we find the point on the number line where the solution set begins. If the number itself is included in the solution set (≤ or ≥) we put a bracket, with an arrow going in either the positive or negative direction. The direction of the arrows will depend upon the inequality. If it is "≤" it will go in the negative direction and if it is "≥" it will go in the positive direction. If the number itself is <u>not</u> in the solution set we put a parenthesis at the number. The direction of the arrow and the direction of the bracket or the parenthesis will depend upon whether the inequality is "<" or ">".

We solve an inequality in much the same way we do an equation.

The **Addition Property of Inequalities** allows us to add the same number to both sides of an inequality without changing the solution set.

$5 > 2$ the left side is greater than the right side.

$$\underbrace{5 + 4}_{9} > \underbrace{2 + 4}_{6}$$

$9 > 6$ the left side is still greater than the right side.

This instructional aid was prepared by the Tallahassee Community College Learning Commons.

$-8 < 4$ the left side is less than the right side.

$\underbrace{-8 + (-2)} < \underbrace{4 + (-2)}$

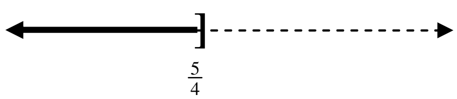
$-10 < 2$ the left side is <u>still</u> less than the right side.

EXAMPLE: Solve: $x - \frac{3}{8} \leq \frac{7}{8}$

$x \underbrace{- \frac{3}{8} + \frac{3}{8}} \leq \frac{7}{8} + \frac{3}{8}$ Add $\frac{3}{8}$ to both sides.

$x + 0 \leq \frac{10}{8}$

$x \leq \frac{5}{4}$

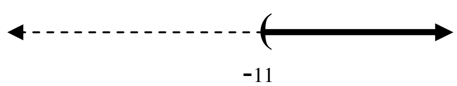


EXAMPLE: $x + 7 > -4$

$x + \underbrace{7 + (-7)} > -4 + (-7)$ Add (-7) to both sides.

$x + 0 > -11$

$x > -11$



The **Multiplication Property of Inequalities** allows us to multiply an inequality by the same **<u>positive</u>** number without changing the solution set.

$6 > 2$ <−−−−the left side is greater than or equal to the right side

$3(6) > 2(3)$ <−−the left side is **<u>still</u>** greater than or equal to the right side

The **Multiplication Property of Inequalities** also says that if we multiply both sides of an inequality by the same **<u>negative</u>** number we must **<u>reverse the inequality symbol</u>** so that the solution set stays the same.

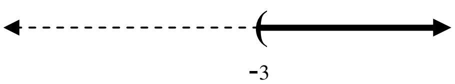
$5 < 8$ <−−− the left side is less than the right side
↓
$5(-2) > 8(-2)$ ***Notice** the inequality symbol was reversed at the time each side was multiplied by the same negative number.

$-10 > -16$ <−−the left side is **<u>no longer less than</u>** the right side
We must reverse the symbol to make the statement true.

SOLVE: $3x > -9$ The coefficient of x is **<u>positive</u>.**

$$\frac{1}{3} \cdot 3x > -9 \cdot \frac{1}{3}$$ Multiply both sides by positive.

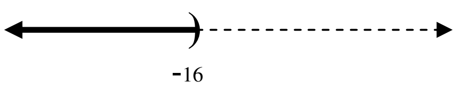
$$x > -3$$ Keep the same symbol.



EXAMPLE: Solve: $(-3/4)x > 12$ (The coefficient of x is **negative)**

$$(-4/3)(-3/4)x < 12\,(-4/3)$$ Multiply both sides by -¾ and **<u>reverse the symbol</u>**

$$x < -16$$



NOTE: We only reverse the inequality symbol if the **<u>coefficient of the variable term is negative.</u>**

<u>EXERCISES</u>:

Solve and graph the solution set.

1. $-6x \leq 18$

2. $y + \frac{5}{9} < \frac{5}{6}$

3. $-\frac{3x}{7} \leq 6$

4. $x + 5 \geq 3$

5. $\frac{2y}{4} > 4$

6. $x - 3 > -2$

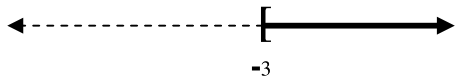
7. $-\frac{3x}{8} < \frac{6}{7}$

8. $\frac{5x}{6} < -20$

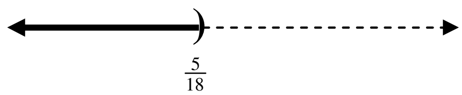
(Answers on next page)

KEYS:

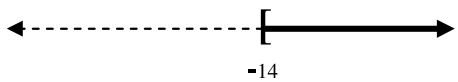
1. $x \geq -3$



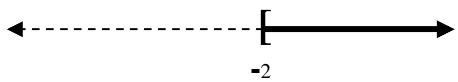
2. $y < \frac{5}{18}$



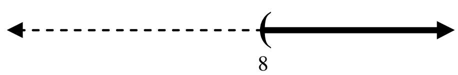
3. $x \geq -14$



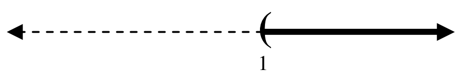
4. $x \geq -2$



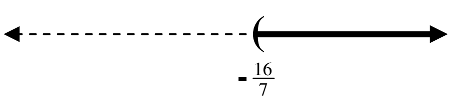
5. $y > 8$



6. $x > 1$



7. $x > -\frac{16}{7}$



8. $x < -24$