Simplifying Variable Expressions By Combining Like Terms

Like terms are terms where the variable parts are <u>identical</u>. This means that the term must have the same letters and that the exponents, if any, must be the same.

$3x$ and $5x$ are like terms.
$3x^2$ and $5x$ are <u>not</u> like terms.

$-3y$ and y are like terms.
$-3y$ and y^3 are <u>not</u> like terms.

To combine like terms we must first identify those terms which are alike. We then combine them by <u>adding the numerical coefficients</u>. The variable part stays the same.

If there is no number with the variable, the numerical coefficient is understood to be 1 or -1 depending upon the sign in front of the terms.

EXAMPLE: $\quad\quad -4x + 9x$
$\quad\quad\quad\quad = (-4 + 9)x \quad\quad\quad\quad$ Use the Distributive Property

The Distributive Property has been used to rewrite "negative four times x" <u>plus</u> "nine times x" as "the sum of negative four and nine" <u>times</u> "x".

$\quad\quad (-4 + 9)x \quad\quad\quad\quad$ Add the numerical coefficients
$\quad\quad = 5x$

EXAMPLE: $\quad\quad m - 8m \quad\quad\quad\quad$ Rewrite subtraction
$\quad\quad\quad\quad m + (-8m)$

Use the Distributive Property to rewrite "one times m <u>plus</u> negative eight times m" to "the sum of one and negative eight times m."

$\quad\quad 1m + (-8m)$
$\quad\quad = \left[1 + (-8)\right]m \quad\quad\quad\quad$ Add the numerical coefficients
$\quad\quad = -7m$

Some variable expressions have more than two terms. These expressions will require us to use the Commutative and Associative Properties of Addition. We may also need to use the Inverse Property of Addition and the Addition Property of Zero in some problems.

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EXAMPLE: $3x^2 + 2x - 7x^2 + 8x$

$3x^2 + 2x - 7x^2 + 8x$ Rewrite subtraction

$3x^2 + (-7x^2) + 2x + 8x$ Use the Commutative property of Addition
                                       to rearrange the terms

$\left[3x^2 + \left(-7x^2\right)\right] + (2x + 8x)$ Use the Associative Property of Addition to
                                       group like terms

$\left[3 + (-7)\right]x^2 + (2+8)x$ Use the Distributive Property

$-4x^2 + 10x$ Add the numerical coefficients

EXAMPLE: $-6y + 2xy + 8y - 2xy$

$-6y + 2xy + 8y - 2xy$ Rewrite the subtraction

$-6y + 8y + 2xy + (-2xy)$ Use the Commutative property of Addition
                                       to rearrange the terms

$(-6y + 8y) + \left[2xy + (-2xy)\right]$ Use the Associative Property of Addition to
                                       group like terms
$(-6 + 8)y + \left[2 + (-2)\right]xy$ Use the Distributive Property

$2y + 0xy$ Use the Inverse property of Addition, then
                                       the multiplication property of zero

$2y$ Use the Addition Property of Zero

A shorter method of combining like terms is to recognize that the sign in front of a term <u>is</u> the sign of that term and the operation is understood to be addition.

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EXAMPLE: $5x - 2y + 3x + 6 - 4y - 9$

$5x + 3x - 2y - 4y + 6 - 9$ As you rearrange the terms this time, take the sign in front of each term with you without rewriting subtraction.

$(5+3)x + [-2 + (-4)]y + [6 + (-9)]$ ADD the coefficients mentally.

$8x - 6y - 3$

The terms of the new polynomial are $8x$, $-6y$ and -3. If we put these together we get the polynomial $8x - 6y - 3$.

NOTE that the sign of each term gives you the addition or subtraction sign between the terms. There must be either a "+"or a "−" between each pair of terms.

EXAMPLE: $-7x^2 + 3x - 2 - 4x^2 + 12x$

$-7x^2 - 4x^2 + 3x + 12x - 2$ Rearrange the terms

$-11x^2 + 15x - 2$ Combine the coefficients mentally

EXERCISES Combine like terms.

1. $-5x - 3x$

2. $8m^2 - 2m + 6m^2$

3. $4 - 8x + 12 - 4x$

4. $-9x^2 - 2x + 8x^2 + x - 7$

5. $12y^3 - 8y^2 - 9y^3 + 8y^2 - y$

6. $\dfrac{3}{5}x^2 - \dfrac{1}{2}x + \dfrac{2}{3}x^2$

KEY:
1. $8x$ 2. $14m^2 - 2m$ 3. $-12x + 16$

4. $-x^2 - x - 7$ 5. $3y^3 - y$ 6. $\dfrac{19}{15}x^2 - \dfrac{1}{2}x$

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