WILLIAM D. LAW, JR.
LEARNING **COMMONS**

Simplifying Square Roots (extended version)

The square root of any positive number is the number that can be squared to get the number whose square root we are seeking. For example,

$\sqrt{16} = 4$ because if we square 4 we get 16, which is the number whose square root is being found.

$\sqrt{64} = 8$ because $8^2 = 64$

$\sqrt{100} = 10$ because $10^2 = 100$

The symbol $\sqrt{}$ is called a radical and it is read as "the square root of."

The number underneath the radical is called the "radicand." In the expression $\sqrt{36}$, the radicand is 36.

It should be noted that each positive number has 2 square roots. One is the positive or *principal* square root, and the other is the *negative* square root.

$\sqrt{4} = 2$ because $2^2 = 4$

$-\sqrt{4} = -2$ because $(-2)^2 = 4$

We are usually interested in the positive square root. If we want the negative root, we put a negative sign in front of the radical.

$$-\sqrt{4} = -2$$
$$-\sqrt{49} = -7$$

Note that we cannot have a negative sign under the radical.

$\sqrt{-4}$ is not a real number, because there is no number that we can multiply by itself and get -4. (In MAT 1033 and MAC 1105 you will learn how to deal with this situation.)

To simplify a radical we must look for and remove any perfect square factors that may be in the radicand. REMEMBER that a perfect square is the square of an integer.

$3^2 = 9$
$4^2 = 16$ Perfect squares
$8^2 = 64$

A radical expression is in simplest form if the radicand contains no perfect square factors. To simplify a radical we will first find the prime factorization of the radicand and rewrite the radicand in exponential form.

$64 = 2^6$ prime factorization of 64 in exponential form.

If the exponent is an even number, then the number itself is a perfect square.

To take the square root of 2^6 we <u>remove the radical and divide the exponent by 2.</u>

EXAMPLE:

$$\sqrt{64} = \sqrt{2^6} = 2^{6/2} = 2^3 = 2 \cdot 2 \cdot 2 = 8$$

Once we have divided the exponent by 2, we can multiply out the remaining factors.

EXAMPLES:

$$\sqrt{81} = \sqrt{3^4} = 3^{4/2} = 3^2 = 3 \cdot 3 = 9$$

$$\sqrt{144} = \sqrt{2^4 \cdot 3^2} = 2^{4/2} \cdot 3^{2/2} = 2^2 \cdot 3 = 4 \cdot 3 = 12$$

$$\sqrt{196} = \sqrt{2^2 \cdot 7^2} = 2^{2/2} \cdot 7^{2/2} = 2 \cdot 7 = 14$$

Often the number we wish to simplify is not a perfect square. We then have to find any perfect square factors contained in the number and remove them from under the radical by taking their square roots. To simplify $\sqrt{40}$, first find the prime factorization of 40.

$$\sqrt{40} = \sqrt{2^3 \cdot 5}$$

NOTICE that the exponents are odd numbers. This means that 2^3 and 5^1 are <u>not</u> perfect squares.

• Any prime factor with an exponent of 1 will not be a perfect square nor will it contain a perfect square.

• Any prime factor with an even exponent will be a perfect square.

• Any prime factor with an odd exponent of 3 or higher will contain a perfect square factor.

$\sqrt{40} = \sqrt{2^3 \cdot 5}$ 2^3 contains a perfect square
$\sqrt{40} = \sqrt{2^2 \cdot 2^1 \cdot 5^1}$ 2^3 is written as $2^2 \cdot 2^1$

The Product Property of Square Roots allows us to rewrite a product under a radical as a product of 2 separate radicals.

$$\sqrt{40} = \sqrt{2^2 \cdot 2^1 \cdot 5^1} = \sqrt{2^2} \cdot \sqrt{2 \cdot 5}$$

We now have on radical which is a perfect square and one which is not. We can take the square root of the perfect square and multiply the factors remaining under the radical.

$$\sqrt{2^2} \cdot \sqrt{2 \cdot 5} = 2\sqrt{10}$$

The complete process is as follows:

$\sqrt{40} = \sqrt{2^3 \cdot 5}$	Find the prime factorization of 40
$= \sqrt{2^2 \cdot 2^1 \cdot 5^1}$	Rewrite 2^3 as $2^2 \cdot 2$
$= \sqrt{2^2} \cdot \sqrt{2 \cdot 5}$	Separate the perfect squares
$= 2\sqrt{10}$	Take square roots. The solution is read as "2 times the square root of 10"

EXAMPLE: Simplify $\sqrt{96}$

$\sqrt{96} = \sqrt{2^5 \cdot 3}$	Find the prime factorization of 96
$= \sqrt{2^4 \cdot 2^1 \cdot 3^1}$	Rewrite 2^5 as $2^4 \cdot 2$
$= \sqrt{2^4} \cdot \sqrt{2 \cdot 3}$	Separate the perfect squares
$= 2^{4/2} \cdot \sqrt{6}$	Take square roots.
$= 2^2\sqrt{6}$	Simplify 2^2 to get 4
$= 4\sqrt{6}$	The solution is read as "4 times the square root of 6"

This instructional aid was prepared by the Tallahassee Community College Learning Commons.

EXAMPLE: Simplify $5\sqrt{180}$. Notice that this is "5 times the square root of 180." We must simplify $\sqrt{180}$ first and then multiply by 5.

$5\sqrt{180} = 5\sqrt{2^2 \cdot 3^2 \cdot 5}$	Find the prime factorization of 180
$= 5\sqrt{2^2 \cdot 3^2} \cdot \sqrt{5}$	Separate the perfect squares
$= 5 \cdot 2^{2/2} \cdot 3^{2/2} \cdot \sqrt{5}$	Take square roots.
$= 5 \cdot 2 \cdot 3 \cdot \sqrt{5}$	Multiply
$= 30\sqrt{5}$	The solution is read as "30 times the square root of 5"

EXAMPLE: Simplify $-8\sqrt{32}$

$-8\sqrt{32} = -8\sqrt{2^5}$	Find the prime factorization of 32
$= -8\sqrt{2^4 \cdot 2}$	Rewrite 2^5 as $2^4 \cdot 2$
$= -8\sqrt{2^4}\sqrt{2}$	Separate the perfect squares
$= -8 \cdot 2^{4/2} \cdot \sqrt{2}$	Take square roots.
$= -8 \cdot 2^2 \cdot \sqrt{2}$	Simplify 2^2 to get 4 and multiply
$= -32\sqrt{2}$	The solution is read as "4 times the square root of 6"

Many of the expressions we will need to simplify will contain variables.

$\sqrt{x^6} = x^3$ because $(x^3)^2 = x^6$

$\sqrt{y^{10}} = y^5$ because $(y^5)^2 = y^{10}$

• Any variable radical expression which has an <u>even</u> exponent will be a perfect square.

$$\sqrt{x^2} = x$$

• Any variable radical expression which has an exponent of 1 will not be a perfect square nor will it contain a perfect square factor.

$$\sqrt{x} = \sqrt{x}$$

• Any variable expression which has an odd exponent of 3 or higher will contain a perfect square factor.

$$\sqrt{x^3} = \sqrt{x^2 \cdot x} = x^{2/2}\sqrt{x} = x\sqrt{x}$$

This instructional aid was prepared by the Tallahassee Community College Learning Commons.

EXAMPLES:

$$\sqrt{x^6} = x^{6/2} = x^3$$

$$\sqrt{x^{18}} = x^{18/2} = x^9$$

$$\sqrt{x^9} = \sqrt{x^8 \cdot x} = x^{8/2}\sqrt{x} = x^4\sqrt{x}$$

We often have radicals which have both numbers and variables.

EXAMPLE: Simplify $\sqrt{45x^2y^3}$

$\sqrt{45x^2y^3} = \sqrt{3^2 \cdot 5 \cdot x^2 \cdot y^3}$	Find the prime factorization of 45
$= \sqrt{3^2 \cdot 5 \cdot x^2 \cdot y^2 \cdot y}$	Rewrite y^3 as $y^2 \cdot y$
$= \sqrt{3^2 \cdot x^2 \cdot y^2}\sqrt{5y}$	Separate the perfect squares and take square roots
$= 3xy\sqrt{5y}$	

EXAMPLE: Simplify $\sqrt{72a^5b^6c^9}$

$\sqrt{72a^5b^6c^9} = \sqrt{2^3 \cdot 3^2 \cdot a^5 \cdot b^6 \cdot c^9}$	Find the prime factorization of 72
$= \sqrt{2^2 \cdot 2 \cdot 3^2 \cdot a^4 \cdot a \cdot b^6 \cdot c^8 \cdot c}$	Rewrite 2^3, a^5, and c^9
$= \sqrt{2^2 \cdot 3^2 \cdot a^4 \cdot b^6 \cdot c^8}\sqrt{2ac}$	Separate the perfect squares and take square roots
$= 2 \cdot 3 \cdot a^2 \cdot b^3 \cdot c^4\sqrt{2ac}$	Multiply the numbers
$= 6a^2b^3c^4\sqrt{2ac}$	

EXAMPLE: Simplify $3y\sqrt{27x^4y^3}$

$3y\sqrt{27x^4y^3} = 3y\sqrt{3^3 \cdot x^4 \cdot y^3}$	Find the prime factorization of 27
$= 3y\sqrt{3^2 \cdot 3 \cdot x^4 \cdot y^2 \cdot y}$	Rewrite 3^3 and y^3
$= 3y\sqrt{3^2 \cdot x^4 \cdot y^2}\sqrt{3y}$	Separate the perfect squares and take square roots
$= 3y \cdot 3 \cdot x^2 \cdot y\sqrt{3y}$	Multiply the numbers and multiply the variables
$= 9x^2y^2\sqrt{3y}$	

This instructional aid was prepared by the Tallahassee Community College Learning Commons.

EXAMPLE: Simplify $4y\sqrt{18x^5y^4}$

$4y\sqrt{18x^5y^4} = 4y\sqrt{2\cdot 3^2\cdot x^5\cdot y^4}$	Find the prime factorization of 18
$= 4y\sqrt{2\cdot 3^2\cdot x^4\cdot x\cdot y^4}$	Rewrite x^5 as $x^4\cdot x$
$= 4y\sqrt{3^2\cdot x^4\cdot y^4}\sqrt{2x}$	Separate the perfect squares and take square roots
$= 4y\cdot 3\cdot x^2\cdot y^2\sqrt{2x}$	Multiply the numbers and multiply the variables
$= 12x^2y^3\sqrt{2x}$	

<u>EXERCISES</u>. Simplify each of the following.

a) $\sqrt{49}$

b) $\sqrt{12}$

c) $\sqrt{40}$

d) $3\sqrt{80}$

e) $\sqrt{x^{14}}$

f) $\sqrt{y^{15}}$

g) $\sqrt{24x^2y^5}$

h) $3\sqrt{20a^5b^2}$

i) $2\sqrt{64x^8y^{10}}$

j) $2xy\sqrt{120x^5y^3}$

<u>KEY</u>

a) 7
b) $2\sqrt{3}$
c) $2\sqrt{10}$
d) $12\sqrt{5}$
e) x^7
f) $y^7\sqrt{y}$
g) $2xy^2\sqrt{6y}$
h) $6a^2b\sqrt{5a}$
i) $16x^4y^5$
j) $4x^3y^2\sqrt{30xy}$

This instructional aid was prepared by the Tallahassee Community College Learning Commons.