Solving Equations With Fractions

When we have an equation which contains fractions, it is easier to solve the equation if we can eliminate the fractions.

EXAMPLE: $\frac{x}{6} = \frac{x}{8} + 9$

This equation will be much easier to solve if we can rewrite an equivalent equation without the fractions.

To eliminate the fractions we must find the Least Common Denominator of all the denominators in the equation.

$$\frac{x}{6} = \frac{x}{8} + \frac{9}{1} \quad \text{(LCD = 24)}$$

We must now multiply **both sides** of the equation by the LCD.

$$\frac{24}{1} \cdot \frac{x}{6} = \frac{24}{1} \cdot \frac{x}{8} + \frac{24}{1} \cdot \frac{9}{1}$$

You can see that this means that **every term** in the equation must be multiplied by 24.

$$24 \cdot \frac{x}{6} = 24 \cdot \frac{x}{8} + 24 \cdot 9$$

We can now cancel common factors and multiply.

$$4x = 3x + 216$$

Now we can solve the new equation. This equation is equivalent to the original one.

$$-3x + 4x = -3x + 3x + 216$$
$$x = 216$$

CHECK: $\frac{x}{6} = \frac{x}{8} + 9$

$$\frac{216}{6} = \frac{216}{8} + 9$$

$$36 = 27 + 9$$

$$36 = 36 \quad \text{TRUE}$$

EXAMPLE: $\frac{5x}{2} + \frac{49}{9} = \frac{12x+7}{9}$

The LCD is 18. We must multiply both sides of the equation by 18.

$$\frac{18}{1}\left(\frac{5x}{2}+\frac{49}{9}\right)=\frac{18}{1}\left(\frac{12x+7}{9}\right)$$

REMEMBER that because we must use the Distributive Property, every **<u>term</u>** must be multiplied by 18.

$$\frac{18}{1}\left(\frac{5x}{2}+\frac{49}{9}\right)=\frac{18}{1}\left(\frac{12x+7}{9}\right)$$ Treat the fraction $\frac{12x+7}{9}$ as one term.

The next step is to cancel common factors and multiply what is left.

$$\frac{18}{1}\left(\frac{5x}{2}\right)+\frac{18}{1}\left(\frac{49}{9}\right)=\frac{18}{1}\left(\frac{12x+7}{9}\right)$$

$45x + 98 = 2(12x + 7)$ There are now 2 terms to multiply by.
$45x + 98 = 24x + 14$

We now have an equivalent equation to solve that does not contain fractions.

$$45x + 98 = 24x + 14$$
$$-24x + 45x + 98 = -24x + 24x + 14$$
$$21x + 98 = 14$$
$$21x + 98 + (-98) = 14 + (-98)$$
$$21x = -84$$
$$\frac{1}{21} \cdot 21x = -84 \cdot \frac{1}{21}$$
$$x = -4$$

CHECK: $\frac{5x}{2} + \frac{49}{9} = \frac{12x+7}{9}$

$$\frac{5(-4)}{2} + \frac{49}{9} = \frac{12(-4)+7}{9}$$

$$\frac{-20}{2} + \frac{49}{9} = \frac{-48+7}{9}$$

$$\frac{-10}{1} + \frac{49}{9} = \frac{-41}{9}$$

$$\frac{-90}{9} + \frac{49}{9} = \frac{-41}{9}$$

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$$\frac{-41}{9} = \frac{-41}{9} \quad \text{TRUE}$$

EXERCISES: Solve and check.

1. $\frac{x}{2} - \frac{x}{5} = 6$

2. $\frac{x}{6} - \frac{x}{9} = \frac{x}{3}$

3. $\frac{x}{8} + \frac{x}{6} = \frac{x}{4} - 1$

4. $\frac{3x-3}{4} = 8 + 2x$

5. $\frac{1}{2}x + \frac{3}{5}x = \frac{1}{10}$

KEY:

1. $x = 20$

2. $x = 0$

3. $x = -24$

4. $x = -7$

5. $x = \frac{1}{11}$