WILLIAM D. LAW, JR. LEARNING COMMONS

A two-tailed Hypothesis Test of a Mean

In testing a hypothesis about a population mean, there are FIVE steps:

1. Identify the claim and Hypotheses.
2. Information and Test Statistic
3. Find the P-value.
4. Interpret the results
5. Write the Conclusion

1. Identify the Claim and write the Null Hypothesis (H_0) and the Alternative Hypothesis (H_1).

Example: Past experience has shown that the scores of an entrance exam are normally distributed with a mean 73. The entrance committee would like to know whether the exam scores of this year's group of 17 applicants are typical. Their average score is 85 with a standard deviation of 9.

H_0: mean $\mu = 73$; this year's applicants are typical. [Claim]

H_1: mean $\mu \neq 73$; this year's applicants are not typical. [Two tail test]

2. Identify the information and calculate the test statistic.

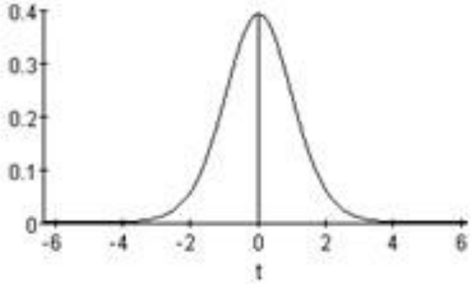
Population mean: $\mu = 73$ sample size: $n = 17$ applicants

Sample mean: x bar = 85 sample standard deviation: $s = 9$

The test statistic is calculated for a t-distribution with n – 1 degrees of freedom. The t-distribution is used because σ is unknown. This means that the standard error, will vary with each sample and it is more likely that more extreme values (values far from 0) will occur than in a standard Normal distribution. The t-distribution helps compensate for that variation.

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} \qquad t = \frac{85-73}{9/\sqrt{17}} \qquad t \approx 5.4975 \text{ with 16 degrees of freedom.}$$

3. Find the p-value; begin by considering the bell shaped t-distribution.



H_0 will be rejected in favor of H_1 if the test average of the applicants is **either significantly higher *or* lower** than the expected score of 73. This makes the test is a two-tail test. Note the "not equal to" symbol in H_1: mean $\mu \neq$ 73.

The p-value in a two-tail test is the total area of both tails measured outward from the center, away from t = 5.4975 or t = -5.4975. To find the p-value, use the **tcdf** function of the Texas Instruments calculator to find the area in one tail and double it.

Press 2^{nd} then VARS select **4: tcdf** press ENTER

The input needed in the tcdf are left bound, right bound, degrees of freedom):

tcdf(5.4975, E99, 16) = 2.43462578 E-5 times 2 = 4.869249156 E-5 approximate to 0.00004869

4. Interpret the test results; compare the p-value with the significance level

The α-value is not given in this problem so assume the significance level is 5% or 0.05.

Since 0.00004868 < 0.05, reject the Null Hypothesis in favor of the Alternative Hypothesis.

5. Write the conclusion in English in the context of the problem.

The exam scores of this year's group of 17 applicants are not typical.

With the calculator:

STAT > TESTS > 2: T-Test > ENTER

This is the calculator input

Inpt: Stats
μ_0: 73
x bar = 85
Sx: 9
n: 17
mean μ: ≠
Calculate:

This is the calculator output

Mean μ ≠ 73
t = 5.497474167
p = 4.8694953E-5
x bar = 85
Sx = 9
n = 17

* **p = 4.8694953E-5**

Remember that the p-value can never be greater than 1. If you see a p-value displayed that appears to be a decimal number greater than 1, look carefully for the E- at the end of the digits and shift the decimal left to include the correct number of zeros