The Z Confidence Interval for a Proportion

Confidence intervals for a proportion give a broader estimate of the population proportion based on the sample proportion. Finding the confidence interval by hand is a multi-step process that involves several formulas. The basic confidence interval formula is:

$$Confidence\ Interval = Point\ Estimate\ \pm Margin\ of\ Error$$

The **point estimate** for a population proportion is the sample proportion, $\hat{p}$. The point estimate is the single value that is the "best guess" of what the population proportion might be. A confidence interval is a range of values that might be reasonable to expect for the population proportion. The margin of error involves the critical value, $Z_{\alpha/2}$, and the standard error. Its formula is:

$$Margin\ of\ Error = Z_{\alpha/2} \times Standard\ Error$$

The critical value depends on what level of confidence is chosen; 90%, 95%, or 98%. Its formula is:

$$Margin\ of\ Error = Z_{\alpha/2} \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

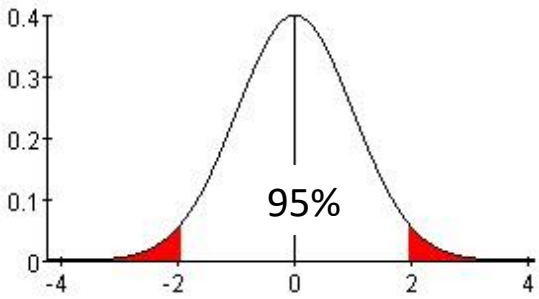
Example:

A group of sociologists want to estimate the true proportion of U.S. citizens with home land lines. They **sample 2563** homes and found out that **752 homes** have land lines. They want to construct the **95% Confidence Interval** of this estimate.

Step 1: Compute the Point Estimate

$$\hat{p} = \frac{752}{2563} \approx 0.2934$$

Step 2: Select a Critical Value:



The area under the normal distribution curve represents the probability of all possible outcomes, which by definition totals 100% (1.00 or simply 1). In this symmetric bell curve, the central, unshaded area represents 95% (0.95) of the distribution leaving 5% (0.05) to be split between the two tails.

An easy way to find the total area to the left of the right tail:

$$Area\ left\ of\ the\ right\ tail = \frac{1 + the\ confidence\ level}{2} \qquad for\ this\ example: \ \frac{1 + .95}{2} = 0.975$$

Use the invNorm function of the Texas Instruments calculator to find the critical value, $Z_{\alpha/2}$

2^{nd} Key > VARS > 3:invNorm > ENTER:

invNorm (Area to the left, mean, standard deviation).

invNorm (0.975,0,1)≈ 1.96

<u>Step 3</u>: Calculate the Margin of Error:

$$M.o.E. = Z_{\alpha_{/2}} \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$M.o.E. = 1.96 \times \sqrt{\frac{0.2934(1-0.2934)}{2563}}$$

$$M.o.E. \approx 0.0176$$

<u>Step 4</u>: Write the Confidence Interval

$$C.I. = 0.2934 \pm 0.0176$$

(0.2934 - 0.0176, 02934 + 0.0176)

(0.2758, 0.311).

<u>Step 4</u>: Interpret the Confidence Interval:

"The Sociologists can be 95% confident that the true proportion of homes with land line phones is between 0.2758 (27.6%) and 0.311(31.1%)."

Use the Texas Instruments calculator to construct the Confidence Interval

Calculator Steps:	**Calculator output:**
STAT > TEST > A: 1-PropZInt:	(.27578, .31103)
x: 752	$\hat{p} = .2934061647$
n: 2563	n = 2563
C-level: .95	
Calculate	