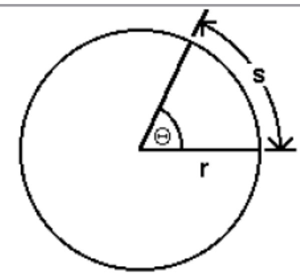
WILLIAM D. LAW, JR. LEARNING **COMMONS**

Finding Angular and Linear Speeds by Dimensional Analysis

i.e. Unit Conversion

Definitions of Symbols		**Relationships**
C = circumference r = radius d = diameter (d = 2r) s = arc length Θ= central angle ***in radians*** π= approximately 3.14 **Formulas** $C = 2\pi r = \pi d$ and $\Theta = \frac{s}{r}$		1 revolution = 1 turn around circle (**definition**) 1 revolution = 2π radians (***angular*** measurement) 1 revolution = $2\pi r$ (***linear*** measurement, i.e. **distance**)
NOTES:	when s = r, Θ= 1 radian	when s = C, Θ= 2π radians

EXAMPLES:

(Note CANCELLATION OF UNITS! Un-cancelled units are ***boldfaced***.)

1. A phonograph record has a radius of 3 inches and revolves at 45 RPM. Find the linear speed of the outside edge of the record.

   Solution: Use the fact that 1 revolution = 2 pi r:

$$\left(\frac{45 \text{ revolutions}}{1\ \textbf{minute}}\right) = \left(\frac{45\ \cancel{\text{revolutions}}}{1\ \textbf{minute}}\right) \cdot \left(\frac{2\pi(3)\ \textbf{inches}}{1\ \cancel{\text{revolution}}}\right) = \left(848.2\ \frac{\text{inches}}{\text{minute}}\right)$$

2. A car is traveling 60 mph. The diameter of the wheels is 3 ft.
   a. Find the number of revolutions per minute the wheels are rotating.

Strategy: We need to convert $\frac{\text{mile}}{\text{hour}} \rightarrow \frac{\text{feet}}{\text{hour}} \rightarrow \frac{\text{feet}}{\text{minute}} \rightarrow \frac{\text{revolutions}}{\text{minute}}$ so, we need three conversion ratios.

$$\left(60\ \frac{\text{miles}}{\text{hour}}\right) = \left(60\ \frac{\cancel{\text{miles}}}{\text{hour}}\right) \cdot \left(\frac{5280 \text{ feet}}{1\ \cancel{\text{mile}}}\right) \qquad \text{(since 1 mile = 5280 feet)}$$

$$\left(60\ \frac{\cancel{\text{miles}}}{\cancel{\text{hour}}}\right) \cdot \left(\frac{5280 \text{ feet}}{1\ \cancel{\text{mile}}}\right) \cdot \left(\frac{1\ \cancel{\text{hour}}}{60\ \textbf{minutes}}\right) \qquad \text{(since 1 hour = 60 minutes)}$$

$$\left(60\ \frac{\cancel{\text{miles}}}{\cancel{\text{hour}}}\right) \cdot \left(\frac{5280\ \cancel{\text{feet}}}{1\ \cancel{\text{mile}}}\right) \cdot \left(\frac{1\ \cancel{\text{hour}}}{60\ \textbf{minutes}}\right) \cdot \left(\frac{1\ \textbf{revolution}}{2\pi(1.5)\ \cancel{\text{feet}}}\right) = \left(560.2\ \frac{\text{revolutions}}{\text{minute}}\right)$$

   b. What's the angular speed of the wheels in radians per minute?

$$\left(560.2\ \frac{\cancel{\text{revolutions}}}{\textbf{minute}}\right) \cdot \left(\frac{2\pi\ \textbf{radians}}{1\ \cancel{\text{revolution}}}\right) = \left(3519.8\ \frac{\text{radians}}{\text{minute}}\right) \qquad \text{(since 1 rev. = } 2\pi \text{ radians)}$$

This instructional aid was prepared by the Tallahassee Community College Learning Commons.