Verifying trigonometric identities

Process: make one side look exactly like the other using a combination of trigonometric identities and algebra. You can work with only one side at a time.

1. Algebra techniques utilized
   a. "FOIL"ing example 1 $(\cot x - \csc x)(\cos x + 1) = -\sin x$
   b. "FOIL"ing example 2 $\dfrac{(\sin t + \cos t)^2}{\sin t \cos t} = 2 + \sec t \csc t$
   c. distribution $\sec t \csc t (\tan t + \cot t) = \sec^2 t + \csc^2 t$
   d. Common denominator $2 \sec x = \dfrac{1}{\sec x + \tan x} + \dfrac{1}{\sec x - \tan x}$
2. Conjugate $\dfrac{1 - \cos x}{\sin x} = \dfrac{\sin x}{1 + \cos x}$
3. Substitution of identity $\sin^2 x + \cos^2 x + \tan^2 x = \sec^2 x$
4. Turn all functions into sin x and cos x $\dfrac{\cos x}{\sec x} + \dfrac{\sin x}{\csc x} = 1$

If all else fails, turn everything into sine x and cosine x and see what happens! Usually there is lots of algebra between using the trig functions. You have to be very familiar with the basic functions.

Basic Functions

$\sec x = \dfrac{1}{\cos x}$ $\csc x = \dfrac{1}{\sin x}$ $\cot x = \dfrac{1}{\tan x}$ $\tan x = \dfrac{\sin x}{\cos x}$

$\sin^2 x + \cos^2 x = 1$ $1 + \cot^2 x = \csc^2 x$ $\tan^2 x + 1 = \sec^2 x$

The last two can be obtained by dividing the first either by sine squared x or cosine squared x. Might also look like cosine x = 1 minus sine squared x or 1 = secant squared x – tangent squared x

Examples

Worked out (remember, work with only one side until it looks like the other)

1. $(\cot x - \csc x)(\cos x + 1) = -\sin x$ ***(working with left side since more complicated)***

$= \cot x \cos x + \cot x - \csc x \cos x - \csc x$ FOIL the binomials

$= \frac{\cos x}{\sin x}\cos x + \frac{\cos x}{\sin x} - \frac{1}{\sin x}\cos x - \frac{1}{\sin x}$ insert sin x /cos x identities

$= \frac{\cos^2 x}{\sin x} + \frac{\cos x}{\sin x} - \frac{\cos x}{\sin x} - \frac{1}{\sin x}$ simplify

$= \frac{\cos^2 x - 1}{\sin x}$ cancel like terms

$= \frac{(1 - \sin^2 x) - 1}{\sin x}$ identity; eliminate cos x term since not in answer

$= \frac{-\sin^2 x}{\sin x} = -\sin x$  reduce

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2. $\frac{(\sin t + \cos t)^2}{\sin t \cos t} = 2 + \sec t \csc t$ ***(working with left side since more complicated)***

$= \frac{\sin^2 t + 2\sin t \cos t + \cos^2 t}{\sin t \cos t}$ FOIL out the top

$= \frac{1 + 2\sin t \cos t}{\sin t \cos t}$ combine $\sin^2 x + \cos^2 x = 1$

$= \frac{1}{\sin t \cos t} + \frac{2\sin t \cos t}{\sin t \cos t}$ separate fraction since final answer doesn't have one

$= \csc t \sec t + 2$  use reciprocals and reduce fraction