**Linear approximation of a function value at a given point.**

**Steps**

1. Find the derivative and substitute the given $x$ value to find $f'(a)$.
2. Substitute the given $x$ value to the original equation to find $f(a)$.
3. Use the equation $y=f\left(a\right)+f'(a)(x-a)$.

**Example 1**

Use the equation $f\left(x\right)=\sqrt{x}$ at $x=4$ to approximate $\sqrt{3.9}$.

Step 1) Find the derivative and substitute the given $x$ value to find $f'(a)$.

$f\left(x\right)=\sqrt{x}$ Original equation

$f\left(x\right)=x^{1/2}$ Write in exponential form.

$f^{'}\left(x\right)=\frac{1}{2}x^{-1/2}$ Use the power rule

$f^{'}\left(x\right)=\frac{1}{2\sqrt{x}}$ Rewrite in radical form.

$f^{'}\left(4\right)=\frac{1}{2\sqrt{4}}$ Substitute $x$ value and solve

$f^{'}\left(4\right)=\frac{1}{4}$ Find $f'(a)$

Step 2) Substitute the given $x$ value to the original equation to find $f(a)$.

$f\left(x\right)= \sqrt{x}$ Original equation

$f\left(4\right)= \sqrt{4}$ Substitute $x$ value and solve

$f\left(4\right)=2$ Find $f(a)$

Step 3) Use the equation $y=f\left(a\right)+f'(a)(x-a)$ where $f(a)$ is the $y$ value of the original equation (2), $f'(a)$ is the $y$ value of the derivative ($\frac{1}{4}$), $x$ is the value you want to approximate (3.9), and $a$ is the value you are using to approximate (4).

$$y=f\left(a\right)+f'(a)(x-a)$$

$$y=2+\frac{1}{4}(3.9-4)$$

$$y=1.975$$

Comparing this with the decimal approximation the calculator gives, it is pretty close.

$$\sqrt{3.9}≈1.9748417$$

**Example 2**

Use the equation $f\left(x\right)=tan⁡(x)$ where $x=\frac{π}{4}$ to approximate $tan⁡(0.8)$.

Step 1) Find the derivative and substitute the given $x$ value to find $f'(a)$.

$f\left(x\right)=tan⁡(x)$ Original equation

$f'\left(x\right)=sec^{2}(x)$ Take derivative

$f^{'}\left(x\right)=sec^{2}(\frac{π}{4})$ Substitute $x$ value and solve

$f^{'}\left(x\right)=2$ Find $f'(a)$

Step 2) Substitute the given $x$ value to the original equation to find $f(a)$.

$f\left(x\right)=tan⁡(x)$ Original equation

$f\left(\frac{π}{4}\right)=tan⁡(\frac{π}{4})$ Substitute $x$ value and solve

$f\left(\frac{π}{4}\right)=1$ Find $f(a)$

Step 3) Use the equation $y=f\left(a\right)+f'(a)(x-a)$ where $f(a)$ is the $y$ value of the original equation (1), $f'(a)$ is the $y$ value of the derivative (2), $x$ is the value you want to approximate (0.8), and $a$ is the value you are using to approximate ($\frac{π}{4}$).

$$y=f\left(a\right)+f'(a)(x-a)$$

$$y=1+2(0.8-\frac{π}{4})$$

$$y≈1.029204$$

Comparing this with the decimal approximation the calculator gives, it is pretty close.

$$tan⁡(0.8)≈1.0296356$$

**Now You Try**

1. Use the equation $f\left(x\right)=\sqrt[3]{x}$ at $x=64$ to approximate$\sqrt[3]{60}$.
2. Use the equation $f\left(x\right)=\frac{1}{x^{2}}$ at $x=5$ to approximate $\frac{1}{24}$.
3. Use the equation $f\left(x\right)=sin⁡(x)$ at $x=π$ to approximate $sin⁡(3)$.

Answers: 1) $3.91\overbar{6}$ 2) 0.041616328 3) 0.141592653